

# Restructuring in Combinatorial Optimization

Mark Sh. Levin

Inst. for Information Transmission Problems, Russian Acad. of Sci.,  
19 Bolshoj Karetny lane, Moscow 127994, Russia  
mslevin@acm.org

**Abstract.** The paper addresses a new class of combinatorial problems which consist in restructuring of solutions (as structures) in combinatorial optimization. Two main features of the restructuring process are examined: (i) a cost of the restructuring, (ii) a closeness to a goal solution. This problem corresponds to redesign (improvement, upgrade) of modular systems or solutions. The restructuring approach is described and illustrated for the following combinatorial optimization problems: knapsack problem, multiple choice problem, assignment problem, spanning tree problems. Examples illustrate the restructuring processes.

**Keywords.** System design, combinatorial optimization, heuristics

## 1 Introduction

The paper addresses a new class of combinatorial problems which are targeted to restructuring of solutions (e.g., a set of elements, a structure) in combinatorial optimization. Two main features of the restructuring process are examined: (i) a cost of the initial problem solution restructuring, (ii) a closeness the obtained restructured solution to a goal solution. Fig. 1 depicts the restructuring process.

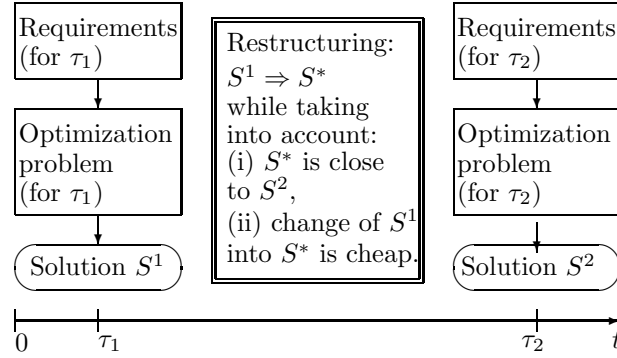


Fig. 1. Illustration for restructuring process

This kind of problems corresponds to redesign/reconfiguration (improvement, upgrade) of modular systems and the situations can be faced in complex software, algorithm systems, communication networks, computer networks, information

systems, manufacturing systems, constructions, etc. ([1], [2], [3], [5], [6], [7], [10]). Here an optimization problem is solved for two time moments:  $\tau_1$  and  $\tau_2$  to obtain corresponding solutions  $S^1$  and  $S^2$ . The examined restructuring problem consists in a “cheap” transformation (change) of solution  $S^1$  to a solution  $S^*$  that is very close to  $S^2$ . This restructuring approach is described and illustrated for the following combinatorial optimization problems (e.g., [4], [6]): knapsack problem, multiple choice problem, assignment problem, spanning tree problems. Numerical examples illustrate the restructuring processes.

## 2 General Restructuring Problems

The restructuring problem may be used for many combinatorial optimization problems as changing a solution (e.g., subset, structure), for example: (i) ranking (sorting) problem, (ii) knapsack problem, (iii) multiple choice problem, (iv) clustering problem, (v) assignment/allocation problems, (vi) bin-packing problem, (vii) graph coloring problem, (viii) vertex covering problems, (ix) clique problem, (x) spanning tree problem, and (xi) Steiner problem. Here it is necessary to take into account a cost of solution changes (e.g., removal of a Steiner node). Fig. 2 illustrates the restructuring problem.

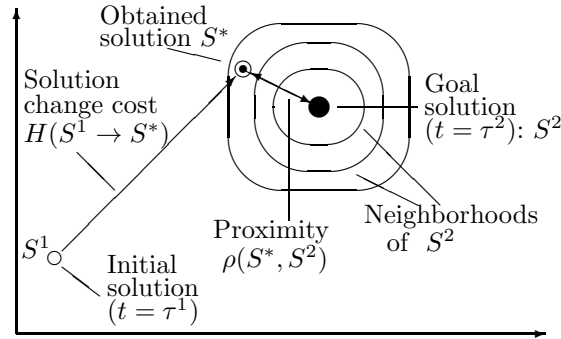


Fig. 2. Illustration for restructuring problem

Let  $P$  be a combinatorial optimization problem with a solution as structure  $S$  (i.e., subset, graph),  $\Omega$  be initial data (elements, element parameters, etc.),  $f(P)$  be objective function(s). Thus  $S(\Omega)$  be a solution for initial data  $\Omega$ ,  $f(S(\Omega))$  be the corresponding objective function. Let  $\Omega^1$  be initial data at an initial stage,  $f(S(\Omega^1))$  be the corresponding objective function.  $\Omega^2$  be initial data at next stage,  $f(S(\Omega^2))$  be the corresponding objective function.

As a result, the following solutions can be considered: (a)  $S^1 = S(\Omega^1)$  with  $f(S(\Omega^1))$  and (b)  $S^2 = S(\Omega^2)$  with  $f(S(\Omega^2))$ . In addition it is reasonable to examine a cost of changing a solution into another one:  $H(S^\alpha \rightarrow S^\beta)$ . Let  $\rho(S^\alpha, S^\beta)$  be a proximity between solutions  $S^\alpha$  and  $S^\beta$ , for example,  $\rho(S^\alpha, S^\beta) = |f(S^\alpha) - f(S^\beta)|$ . Note function  $f(S)$  is often a vector function. Finally, the restructuring problem can be examine as follows (a basic version):

Find a solution  $S^*$  while taking into account the following:  
(ii)  $H(S^1 \rightarrow S^*) \rightarrow \min$ , (ii)  $\rho(S^*, S^2) \rightarrow \min$  (or constraint).

Thus the basic optimization model can be examined as the following:

$$\min \rho(S^*, S^2) \quad s.t. \quad H(S^1 \rightarrow S^*) \leq \hat{h},$$

where  $\hat{h}$  is a constraint for cost of the solution change. Fig. 3 illustrates restructuring of a multicriteria problem. Note proximity function  $\rho(S^*, S^2)$  (or  $\rho(S^{*j}, \{S^{21}, S^{22}, S^{23}\})$  can be considered as a vector function as well (analogically for the solution change cost). This situation will lead to a multicriteria restructuring problem.

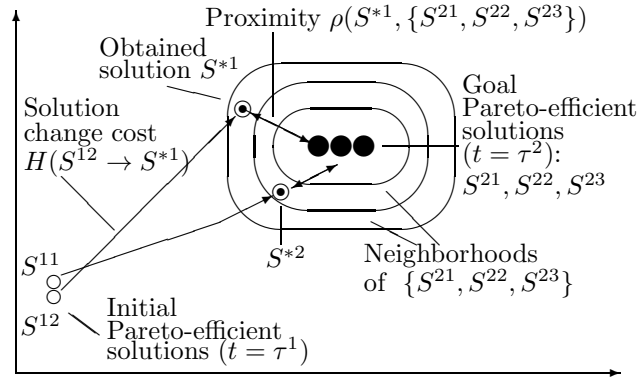


Fig. 3. Restructuring of multicriteria problem

### 3 Restructuring in Some Combinatorial Problems

Let  $A = \{1, \dots, i, \dots, n\}$  be an initial set of elements. Knapsack problem is considered for two time moments  $\tau_1$  and  $\tau_2$  (for  $\tau_2$  parameters  $\{c_i^2\}$ ,  $\{a_i^2\}$ , and  $b^2$  are used) (Fig. 4):

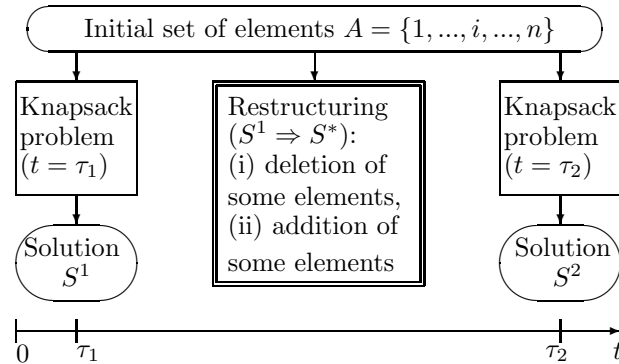


Fig. 4. Restructuring in knapsack problem

$$\max \sum_{i=1}^n c_i^1 x_i \quad s.t. \quad \sum_{i=1}^n a_i^1 x_i \leq b^1, \quad x_i \in \{0, 1\}.$$

The corresponding solutions are:  $S^1 \subseteq A$  ( $t = \tau_1$ ) and  $S^2 \subseteq A$  ( $t = \tau_2$ ) ( $S^1 \neq S^2$ ).

**Illustrative numerical example:**  $A = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $S^1 = \{1, 3, 4, 5\}$ ,  $S^2 = \{2, 3, 5, 7\}$ ,  $S^* = \{2, 3, 4, 6\}$ . The change (restructuring) process (i.e.,  $S^1 \Rightarrow S^*$ ) is based on the following (Fig. 5): (a) deleted elements:  $S^{1*-} = S^1 \setminus S^* = \{1, 5\}$ , (b) added elements:  $S^{1*+} = S^* \setminus S^1 = \{2, 6\}$ .

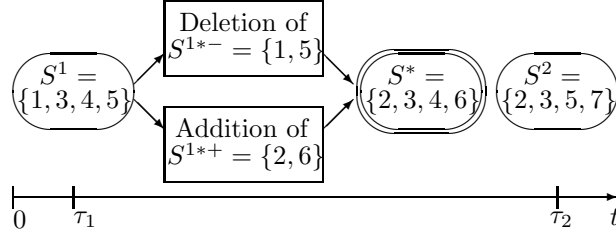


Fig. 5. Example for restructuring

Note the following exists at the start stage of the solving process:  $S^{1*-} = S^1$  and  $S^{1*+} = A \setminus S^1$ . The restructuring problem can be considered as the following:

$$\min \rho(S^*, S^2) \quad s.t. \quad H(S^1 \Rightarrow S^*) = \left( \sum_{i \in S^{1*-}} h_i^- + \sum_{i \in S^{1*+}} h_i^+ \right) \leq \hat{h}, \quad \sum_{i \in S^*} a_i^2 \leq b^2,$$

where  $\hat{h}$  is a constraint for the change cost,  $h^-(i)$  is a cost of deletion of element  $i \in A$ , and  $h^+(i)$  is a cost of addition of element  $i \in A$ . On the other hand, an equivalent problem can be examined:

$$\max \sum_{i \in S^*} x_i c_i^2 \quad s.t. \quad H(S^1 \Rightarrow S^*) = \left( \sum_{i \in S^{1*-}} h_i^- + \sum_{i \in S^{1*+}} h_i^+ \right) \leq \hat{h}, \quad \sum_{i \in S^*} a_i^2 \leq b^2,$$

because  $\max \sum_{i \in S^*} x_i c_i^2 \leq \max \sum_{i \in S^2} x_i c_i^2$  while taking into account constraint:  $\sum_{i \in S^*} a_i^2 \leq b^2$ . The obtained problem is a modified knapsack-like problem as well. At the same time, it is possible to use a simplified solving scheme (by analysis of *change elements* for addition/deletion): (a) generation of candidate elements for deletion (i.e., selection of  $S^{1*-}$  from  $S^1$ ), (b) generation of candidate elements for addition (i.e., selection of  $S^{1*+}$  from  $A \setminus S^1$ ). The selection processes may be based on multicriteria ranking. As a result, a problem with sufficiently decreased dimension will be obtained.

Basic multiple choice problem is for  $t = \tau_1$  (for  $t = \tau_2$  parameters  $\{c_{ij}^2\}$ ,  $\{a_{ij}^2\}$ , and  $b^2$  are used):

$$\max \sum_{i=1}^m \sum_{j=1}^{q_i} c_{ij}^1 x_{ij} \quad s.t. \quad \sum_{i=1}^m \sum_{j=1}^{q_i} a_{ij}^1 x_{ij} \leq b^1, \quad \sum_{j=1}^{q_i} x_{ij} \leq 1 \quad \forall i = \overline{1, m}, \quad x_{ij} \in \{0, 1\}.$$

Here initial element set  $A$  is divided into  $m$  subsets (without intersection):  $A = \bigcup_{i=1}^m A_i$ , where  $A_i = \{1, \dots, j, \dots, q_i\}$  ( $i = \overline{1, m}$ ). Thus each element is denoted by  $(i, j)$ . An equivalent problem is:

$$\max \sum_{(i,j) \in S^1} c_{ij}^1 \quad s.t. \quad \sum_{(i,j) \in S^1} a_{ij}^1 \leq b^1, \quad |S^1 \& A_i| \leq 1 \quad \forall i = \overline{1, m}.$$

For  $t = \tau_2$  the problem is the same.

**Illustrative numerical example:**  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ ,  $A_1 = \{1, 3, 5, 12\}$ ,  $A_2 = \{2, 7, 9\}$ ,  $A_3 = \{4, 8, 13\}$ ,  $A_4 = \{6, 10, 11\}$ ,  $S^1 = \{1, 7, 8, 11\}$ ,  $S^2 = \{3, 7, 8, 10\}$ ,  $S^* = \{1, 2, 8, 6\}$ . The change (restructuring) process (i.e.,  $S^1 \Rightarrow S^*$ ) is based on the following (Fig. 6): (a) deleted elements:  $S^{1*-} = S^1 \setminus S^* = \{7, 11\}$ , (b) added elements:  $S^{1*+} = S^* \setminus S^1 = \{2, 6\}$ .

Thus the restructuring problem can be considered as the following:

$$\begin{aligned} & \min \rho(S^*, S^2) \\ & s.t. \quad H(S^1 \Rightarrow S^*) = \left( \sum_{(i,j) \in S^{1*-}} h_{ij}^- + \sum_{(i,j) \in S^{1*+}} h_{ij}^+ \right) \leq \hat{h}, \quad \sum_{(i,j) \in S^*} a_{ij}^2 \leq b^2, \\ & \quad |S^* \& A_i| \leq 1 \quad \forall i = \overline{1, m}. \end{aligned}$$

where  $\hat{h}$  is a constraint for the change cost,  $h^-(ij)$  is a cost of deletion of element  $(i, j) \in A$ , and  $h^+(ij)$  is a cost of addition of element  $(i, j) \in A$ . An equivalent problem is:

$$\begin{aligned} & \max \sum_{(i,j) \in S^*} c_{ij}^2 \\ & s.t. \quad H(S^1 \Rightarrow S^*) = \left( \sum_{(i,j) \in S^{1*-}} h_{ij}^- + \sum_{(i,j) \in S^{1*+}} h_{ij}^+ \right) \leq \hat{h}, \quad \sum_{(i,j) \in S^*} a_{ij}^2 \leq b^2, \\ & \quad |S^* \& A_i| \leq 1 \quad \forall i = \overline{1, m}. \end{aligned}$$

The simplest version of algebraic assignment problem is:

$$\max \sum_{i=1}^m \sum_{j=1}^n c_{i,j}^1 x_{i,j} \quad s.t. \quad \sum_{i=1}^m x_{i,j} \leq 1, j = \overline{1, n}; \quad \sum_{j=1}^n x_{i,j} \leq 1, i = \overline{1, m}; \quad x_{i,j} \in \{0, 1\}.$$

This problem is polynomially solvable. Let us consider  $n = m$ . Thus a solution can be examined as a permutation of elements  $A = \{1, \dots, i, \dots, n\}$ :  $S = \langle s[1], \dots, s[i], \dots, s[n] \rangle$ , where  $s[i]$  defines the position of element  $i$  in the resultant permutation  $S$ . Let  $c(i, s[i]) \geq 0$  ( $i = \overline{1, n}$ ) be a “profit” of assignment of element  $i$  into position  $s[i]$  (i.e.,  $\|c(i, s[i])\|$  is a “profit” matrix).

The combinatorial formulation of assignment problem is:

$$\text{Find permutation } S \text{ such that } \sum_{i=1}^n c(i, s[i]) \rightarrow \max.$$

Now let us consider three solutions (permutations):

- (a)  $S^1 = \langle s^1[1], \dots, s^1[i], \dots, s^1[n] \rangle$  for  $t = \tau_1$ ,
- (b)  $S^2 = \langle s^2[1], \dots, s^2[i], \dots, s^2[n] \rangle$  for  $t = \tau_2$ , and
- (c)  $S^* = \langle s^*[1], \dots, s^*[i], \dots, s^*[n] \rangle$  (the restructured solution).

**Illustrative numerical example:**  $A = \{1, 2, 3, 4, 5, 6, 7\}$ ,

$S^1 = \{2, 4, 5, 1, 3, 7, 6\}$ ,  $S^2 = \{4, 1, 3, 7, 5, 2, 6\}$ ,  $S^* = \{2, 4, 3, 1, 5, 7, 6\}$ .

Here the following changes are made in  $S^1$ :  $5 \rightarrow 3$ ,  $3 \rightarrow 5$ . Clearly, the changes can be based on typical exchange operations: *2-exchange*, *3-exchange*, etc.

Further, let us consider a vector of structural difference (by components) for two permutations  $S^\alpha$  and  $S^\beta$ :  $\{s^\alpha[i] - s^\beta[i], i = \overline{1, n}\}$  and a change cost matrix  $\|d(i, j)\|$  ( $i = \overline{1, n}, j = \overline{1, n}$ ). Here  $d(i, i) = 0 \quad \forall i = \overline{1, n}$ . Evidently, the cost for restructuring solution  $S^1$  into solution  $S^*$  is:  $H(S^1 \rightarrow S^*) = \sum_{i=1}^n h(s^1[i], s^*[i])$ . Proximity (by “profit”) for two permutations  $S^\alpha$  and  $S^\beta$  may be considered as follows:  $\rho(S^\alpha, S^\beta) = |\sum_{i=1}^n c^\alpha(i, s^\alpha[i]) - \sum_{i=1}^n c^\beta(i, s^\beta[i])|$ . Finally, the restructuring of assignment is (a simple version):

$$\min \rho(S^*, S^2) \quad s.t. \quad H(S^1 \rightarrow S^*) = \sum_{i=1}^n h(s^1[i], s^*[i]) \leq \hat{h}.$$

Restructuring problems for minimal spanning tree problem and for Steiner tree problem are described as follows (Fig. 6, Fig. 7). The following numerical examples are presented:

**I.** Initial graph (Fig. 6):  $G = (A, E)$ , where  $A = \{1, 2, 3, 4, 5, 6, 7\}$ ,  
 $E = \{(1, 2), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (3, 6), (4, 5), (4, 6), (5, 6), (5, 7), (6, 7)\}$ .

**II.** Spanning trees (Fig. 6):

- (i)  $T^1 = (A, E^1)$ , where  $E^1 = \{(1, 2), (1, 4), (1, 6), (3, 5), (5, 6), (6, 7)\}$ ,
- (ii)  $T^2 = (A, E^2)$ , where  $E^2 = \{(1, 2), (2, 3), (2, 6), (4, 6), (5, 6), (6, 7)\}$ ,
- (iii)  $T^* = (A, E^*)$ , where  $E^* = \{(1, 2), (1, 4), (2, 3), (2, 6), (3, 5), (6, 7)\}$ .

Here the edge changes are ( $T^1 \rightarrow T^*$  as  $E^1 \rightarrow E^*$ ):

$E^{1*-} = \{(1, 6), (5, 6)\}$  and  $E^{1*+} = \{(2, 3), (2, 6)\}$ .

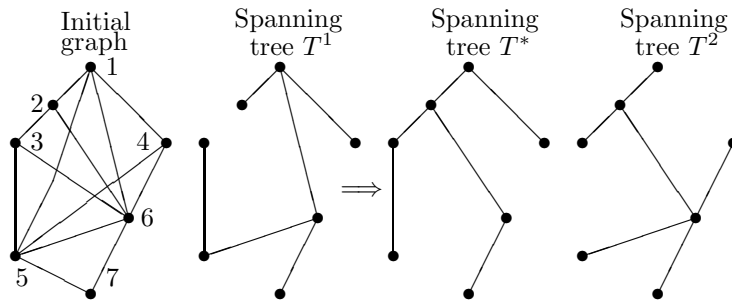


Fig. 6. Restructuring of spanning tree

**III.** Steiner trees (Fig. 7, set of possible Steiner vertices is  $Z = \{a, b, c, d\}$ ):

- (i)  $S^1 = (A^1, E^1)$ , where  $A^1 = A \cup Z^1$ ,  $Z^1 = \{a, b\}$ ,

- $E^1 = \{(1, 2), (1, a), (a, 4), (a, 6), (3, 5), (b, 5), (b, 6), (b, 7)\},$   
(ii)  $S^2 = (A^2, E^2)$ , where  $A^2 = A \cup Z^2$ ,  $Z^2 = \{a, b, d\},$   
 $E^2 = \{(3, 4), (1, d), (3, d), (a, d), (a, 4), (a, 6), (b, 6), (b, 5), (b, 7)\},$   
(iii)  $S^* = (A^*, E^*)$ , where  $A^* = A \cup Z^*$ ,  $Z^* = \{a, c\},$   
 $E^* = \{(1, 2), (1, a), (a, 4), (a, 6), (c, 3), (c, 5), (c, 6), (6, 7)\}.$

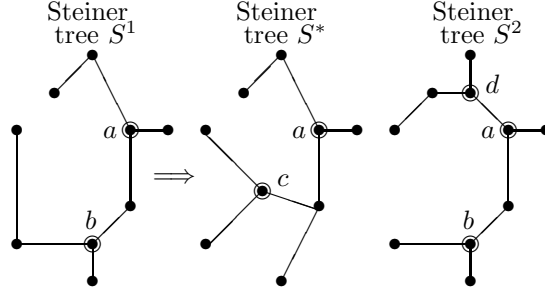


Fig. 7. Restructuring of Steiner tree

Thus the restructuring problem for spanning tree is (Fig. 6, a simple version):

$$\min \rho(T^*, T^2) \quad s.t. \quad H(S^1 \Rightarrow S^*) = \left( \sum_{i \in E^{1*-}} h_i^- + \sum_{i \in E^{1*+}} h_i^+ \right) \leq \hat{h},$$

where  $\hat{h}$  is a constraint for the change cost,  $h^-(i)$  is a cost of deletion of element (i.e., edge)  $i \in E^1$ , and  $h^+(i)$  is a cost of addition of element (i.e., edge)  $i \in E \setminus E^1$ .

The restructuring problem for Steiner tree is (Fig. 7, a simple version):

$$\min \rho(S^*, S^2)$$

$$s.t. \quad H(S^1 \Rightarrow S^*) = \left( \sum_{i \in E^{1*-}} h_i^- + \sum_{i \in E^{1*+}} h_i^+ \right) + \left( \sum_{i \in Z^{1*-}} w_i^- + \sum_{i \in Z^{1*+}} w_i^+ \right) \leq \hat{h},$$

where  $\hat{h}$  is a constraint for the change cost,  $h^-(i)$  is a cost of deletion of element (i.e., edge)  $i \in E^1$ ,  $h^+(i)$  is a cost of addition of element (i.e., edge)  $i \in \hat{E}^* \subseteq E \setminus E^1$ ,  $w^-(j)$  is a cost of deletion of Steiner vertex  $j \in Z^1$ ,  $w^+(j)$  is a cost of addition of Steiner vertex  $j \in \hat{Z}^* \subseteq Z \setminus Z^1$ .

In the main, the suggested restructuring problems are NP-hard and enumerative algorithms or heuristics can be used. The design/selection of heuristics may be based on some typical situations. First, the restructuring problems often are based on two selection subproblems: (a) deletion of elements and (b) addition of elements. This leads to possible usage of greedy-like algorithms. If the restructuring problem is based on exchange of elements (e.g., restructuring in assignment/allocation problem) local heuristics as k-exchange techniques can be used (e.g., 2-OPT, 3-OPT for travelling salesman problems). Further, methods of constraint programming can be widely used. Evidently, many well-known meta-heuristic methods can be used as well. In addition, heuristic can be based on reducing of the basic restructuring problem, for example: (a) by problem type, (b) by problem dimension (e.g, selection of the most prospective change operations), etc.

## 4 Illustrative Application Examples

**Example 1.** Reconfiguration of “microelectronic components part” in wireless sensor (multiple choice problem)  $M = R \star P \star D \star Q$  [8]:

1. Radio  $R$ : 10 mw 916 MHz Radio  $R_1(3)$ , 1 mw 916 MHz Radio  $R_2(2)$ , 10 mw 600 MHz Radio  $R_3(2)$ , 1 mw 600 MHz Radio  $R_4(1)$ .
2. Microprocessor  $P$ : MAXQ 2000  $P_1(1)$ , AVR with embedded DAC/ADC  $P_2(2)$ , MSP  $P_3(3)$ .
3. DAC/ADC  $D$ : Motorola  $D_1(2)$ , AVR embedded DAC/ADC  $D_2(1)$ , Analog Devices 1407  $D_3(2)$ .
4. Memory  $Q$ : 512 byte RAM  $Q_1(3)$ , 512 byte EEPROM  $Q_2(3)$ , 8 KByte Flash  $Q_3(2)$ , 1 MByte Flash  $Q_4(1)$ .

Table 1. Estimates of DAs

	Cost ( $a_{ij}$ )	Change cost		Priorities	
		$h_{ij}^-$	$h_{ij}^+$	$c_{ij}^1$	$c_{ij}^2$
$R_1$	6	2	2	1	1
$R_2$	5	1	1	2	3
$R_3$	3	2	1	2	1
$R_4$	2	2	2	3	2
$P_1$	5	2	3	3	2
$P_2$	10	2	2	2	3
$P_3$	30	3	2	1	2
$D_1$	2	2	3	2	3
$D_2$	1	2	2	3	2
$D_3$	2	1	1	2	1
$Q_1$	3	2	1	1	3
$Q_2$	2	2	2	1	3
$Q_3$	3	1	2	2	2
$Q_4$	3	1	1	3	2

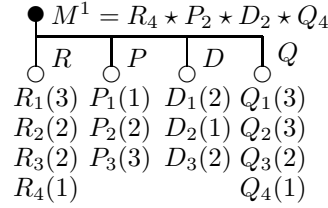


Fig. 8. Structure of  $M^1$

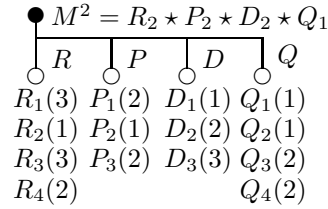


Fig. 9. Structure of  $M^2$

Here it is assumed that solutions are based on multiple choice problem (in [8] the solving process was based on morphological clique problem while taking into account compatibility of selected DAs). Thus two solutions  $M^1$  (for  $t = \tau_1$ , Fig. 8) and  $M^2$  (for  $t = \tau_2$ , Fig. 9) are examined (in [8] the solutions correspond to trajectory design: stage 1 and stage 3). Table 1 contains estimates of DAs (expert judgment). Estimates of cost (Table 1) and priorities (Fig. 8, Fig. 9, in parentheses) correspond to examples in [8]. Here  $c_{ij} = 4 - p_{ij}$ . Two possible change operations can be considered ( $M^1 \Rightarrow M^*$ ,  $M^*$  is close to  $M^2$ ):

- (a)  $R_4 \rightarrow R_2$ ,  $h_a^- = 2$ ,  $h_a^+ = 1$  (corresponding Boolean variable  $x_a \in \{0, 1\}$ ),
- (b)  $Q_4 \rightarrow Q_1$ ,  $h_b^- = 1$ ,  $h_b^+ = 1$  (corresponding Boolean variable  $x_b \in \{0, 1\}$ ).

As a result, the following simplified knapsack problem can be used:

$$\begin{aligned} \max \quad & (c^2(R_2) - c^2(R_4)) x_a + (c^2(Q_1) - c^2(Q_4)) x_b \\ \text{s.t.} \quad & H(M^* \rightarrow M^2) = (h^-(R_4 \rightarrow R_2) + h^+(R_4 \rightarrow R_2)) x_a + \end{aligned}$$



$$(h^-(Q_4 \rightarrow Q_1) + h^+(Q_4 \rightarrow Q_1)) x_b \leq \hat{h}.$$

Finally, the restructuring solutions are: (i)  $\hat{h} = 2$ :  $M^{*1} = R_4 \star P_2 \star D_2 \star Q_1$ , (ii)  $\hat{h} = 3$ :  $M^{*2} = R_2 \star P_2 \star D_2 \star Q_4$ , (iii)  $\hat{h} = 5$ :  $M^{*3} = M^2 = R_2 \star P_2 \star D_2 \star Q_1$ . Evidently, real restructuring problems can be more complicated.

**Example 2.** Reassignment of users to access points ([7], [9]). Here the initial multicriteria assignment problem involves 21 users and 6 access points. Tables 2, 4 contain some parameters for users ( $A$ ) (coordinates  $(x_i, y_i, z_i)$ , required frequency spectrum  $f_j$ , required level of reliability  $r_j$ , etc.) and some parameters for 6 access points ( $B = \{j\} = \{1, 2, 3, 4, 5, 6\}$ ) (coordinates  $(x_j, y_j, z_j)$ , frequency spectrum  $f_j$ , number of connections  $n_j$ , level of reliability  $r_j$ ) ([7], [9]). A simplified version of assignment problem from [7] is considered. Two regions are examined: an initial region and an additional region (Fig. 10). In [7] the problem was solved for two cases: (i) separated assignment  $S^1$  (Fig. 10), (ii) joint assignment  $S^2$  (Fig. 11). The restructured problem is considered as a modification (change) of  $S^1$  into  $S^*$ . To reduce the problem it is reasonable to select a subset of users (a “change zone” near borders between regions):  $\tilde{A} = \{i\} = \{3, 5, 8, 12, 13, 14, 17, 19, 21\}$ . Thus, it is necessary to assign each element of  $\tilde{A}$  into an access point of  $B$ .

Table 2. Access points

$j$	$x_j$	$y_j$	$z_j$	$f_j$	$n_j$	$r_j$
1	50	157	10	30	4	10
2	72	102	10	42	6	10
3	45	52	10	45	10	10
4	150	165	10	30	5	15
5	140	112	10	32	5	8
6	147	47	10	30	5	15

Table 3. Users-access points

$i$	Access points $\{j\}$ : $h_{i,j}^-, h_{i,j}^+, c_{i,j}$					
	1	2	3	4	5	6
3	3, 2, 2	2, 1, 3	1, 0, 3	3, 1, 3	2, 1, 0	1, 1, 0
5	2, 1, 1	1, 3, 1	1, 2, 1	3, 2, 1	1, 1, 1	1, 1, 1
8	1, 1, 3	1, 1, 3	1, 1, 3	1, 1, 0	1, 1, 3	2, 2, 2
12	2, 2, 3	1, 2, 3	1, 2, 3	3, 1, 0	2, 1, 0	1, 1, 0
13	1, 1, 3	1, 1, 3	1, 1, 3	2, 1, 0	2, 2, 1	1, 1, 3
14	1, 1, 1	2, 2, 2	1, 2, 0	1, 1, 1	1, 1, 1	1, 1, 0
17	1, 1, 2	1, 1, 1	1, 0, 1	3, 1, 1	1, 1, 1	1, 1, 1
19	1, 1, 0	1, 1, 3	1, 2, 3	3, 2, 0	1, 1, 3	1, 1, 2
21	1, 1, 0	1, 2, 3	1, 1, 2	3, 1, 1	1, 1, 1	1, 1, 1

The considered simplified restructuring problem is based on set of change operations: (1) user 3, change of connection:  $1 \rightarrow 4$  (Boolean variable  $x_1$ ), (2) user 13, change of connection:  $3 \rightarrow 6$  (Boolean variable  $x_2$ ), (3) user 21, change of connection:  $5 \rightarrow 2$  (Boolean variable  $x_3$ ). Table 3 contains estimates of change costs (expert judgment) and “integrated profits” of correspondence between users and access points from ([7], [9]). The problem is:

$$\max (c_{3,4} x_1 + c_{13,6} x_2 + c_{21,2} x_3)$$

$$s.t. ( (h_{3,1}^- + h_{3,4}^+) x_1 + (h_{13,3}^- + h_{13,6}^+) x_2 + (h_{21,5}^- + h_{21,2}^+) x_3 ) \leq \hat{h}.$$

The reassignment  $S^*$  is depicted in Fig. 12 (i.e.,  $x_1 = 0, x_2 = 1, x_3 = 1, \hat{h} = 5$ ).

Table 4. Users

$i$	$x_i$	$y_i$	$z_i$	$f_i$	$r_i$
1	30	165	5	10	5
2	58	174	5	5	9
3	95	156	0	6	6
4	52	134	5	6	8
5	85	134	3	6	7
6	27	109	7	8	5
7	55	105	2	7	10
8	98	89	3	10	10
9	25	65	2	7	5
10	52	81	1	10	8
11	65	25	7	6	9
12	93	39	1	10	10
13	172	26	2	10	7
14	110	169	5	7	5
15	145	181	3	5	4
16	150	150	5	7	4
17	120	140	6	4	6
18	150	136	3	6	7
19	135	59	4	13	4
20	147	79	5	7	16
21	127	95	5	7	5

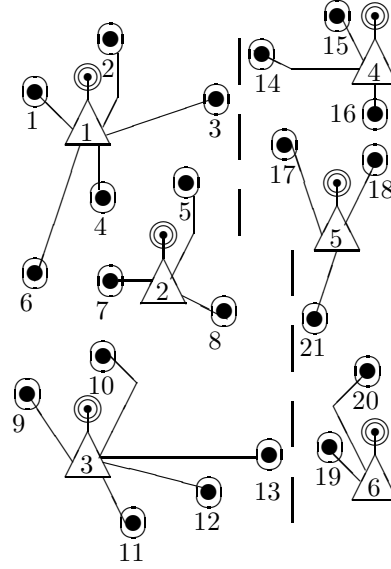


Fig. 10. Separated assignment  $S^1$

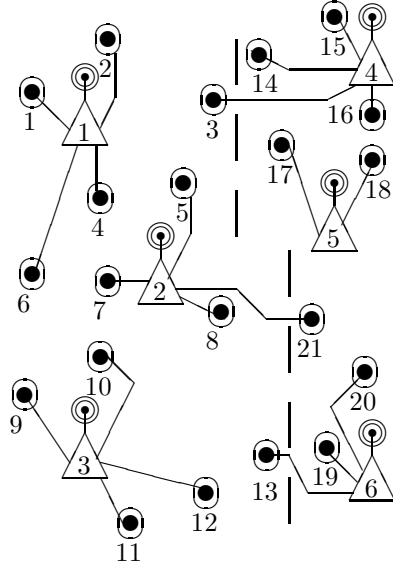


Fig. 11. Joint assignment  $S^2$

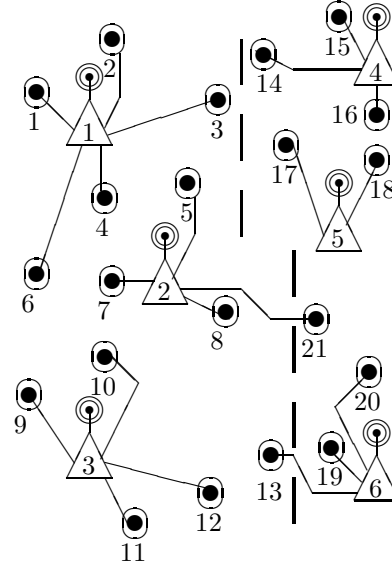


Fig. 12. Joint assignment  $S^*$

## 5 Conclusion

In the paper a restructuring approach in combinatorial optimization is suggested. The restructuring problem is formulated as a combinatorial optimization problem with one objective function. Multicriteria problem statement is briefly described as well. The restructuring approach is applied for several combinatorial optimization problems (knapsack problem, multiple choice problem, assignment problem, minimum spanning tree, Steiner tree problem). Some application domains are pointed out (e.g., sensors, communication networks). The suggested restructuring approach is the first step in this research field. Clearly, it is reasonable to consider other types of system reconfiguration problems. In the future it may be prospective to consider the following research directions: 1. application of the suggested restructuring approach to other combinatorial optimization problems (e.g., covering, graph coloring); 2. examination of multicriteria restructuring models; 3. examination of restructuring problems with changes of basic element sets (i.e.,  $A^1 \neq A^2$ ); 4. study and usage of various types of proximity between obtained solution(s) and goal solution(s) (i.e.,  $\rho(S^*, S^2)$ ); 5. examination of the restructuring problems under uncertainty (e.g., stochastic models, fuzzy sets based models); 6. reformulation of restructuring problem(s) as satisfiability model(s); 7. usage of various AI techniques in solving procedures; and 8. application of the suggested restructuring approaches in engineering/CS education.

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